

# A Lyapunov treatment of swarm coordination under conflict

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### Abstract

We consider hostile conflicts between two multi-agent swarms, called pursuers and evaders. A Newtonian dynamicsbased double integrator model is taken into account, as well as a control strategy using the relative positions and velocities of opposing swarm members. This control is introduced to achieve stability and the capture of the evaders by the pursuers. The present document considers only swarms with equal membership strengths and equal mass for simplicity. This effort begins with a set of suggested interaction force profiles, which are functions of local vectors. To formulate a robust control law, a Lyapunov-based stability analysis is used. The group pursuit is conceived in two phases: the *approach phase*, during which the two swarms act like two individual agents, and the *assigned pursuit phase*, where each pursuer has an assigned evader. We show that the uncontrolled dynamics, which are marginally stable, are stabilized by the new controller.

#### **Keywords**

Lyapunov stability, multi-agent modeling, stability of nonlinear systems

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## I. Introduction

This study addresses the modeling, analysis and control of multi-agent swarms starting from Newton's equations. When each agent influences every other agent, the rules of interaction and stability of group dynamics become quite complicated. Much of the inspiration stems from the study of biological swarms (Warburton and Lazarus, 1991; Camazine et al., 2001). Most of the earlier investigations that focus on this general theme consider homogenous swarms, i.e., those composed of alike members, with a single integrator model and momenta profiles (Chu et al., 2003, 2006; Gazi and Passino, 2003, 2004a, 2004b; Yao et al., 2007). A difference in treatment in this paper is to start from Newton's second law instead.

Gazi and Passino (2004a) expand an earlier pioneering work on swarm coordination by generalizing the stability analysis for a class of attraction/repulsion functions for homogenous swarms. As a consequence of the homogenous membership and symmetric characteristics of the momenta, stationary and stable swarm behavior is achieved. They also propose modifications to account for finite body size of the swarm members. In these studies, all members are required to know (or sense) the position of all the others.

Chu et al. (2003, 2006) address the stability analysis of anisotropic (asymmetric behavior), but nonhostile, swarms. They propose some aggregation rules for swarms with reciprocal and nonreciprocal interactions between agents. They also point out that the swarming behavior results from interplay between long-range attraction and short-range repulsion among individuals. For nonreciprocal interaction, a condition of weighted momenta is assumed (Chu et al., 2003). In the present paper, we extend the application of asymmetric

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